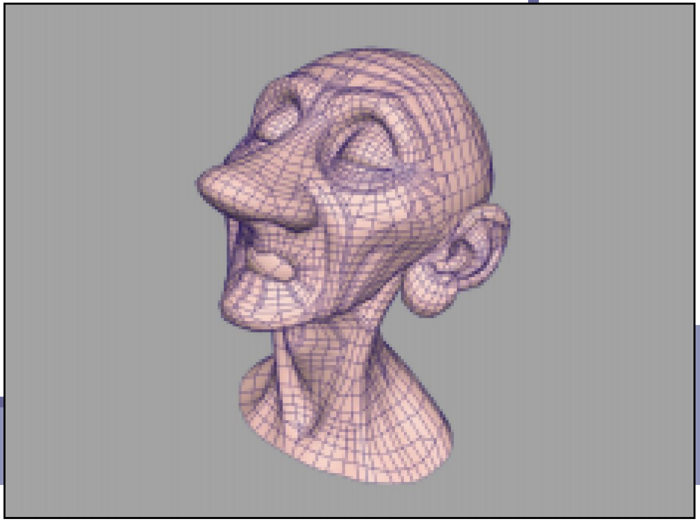
Further Graphics

Subdivision Surfaces



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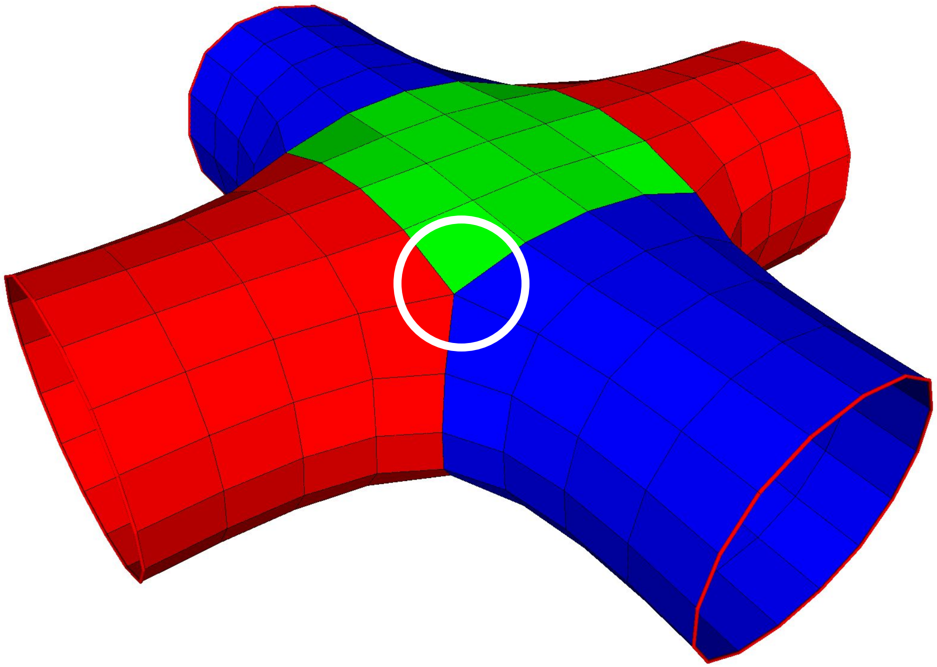
Problems with Bezier (NURBS) patches

- Joining spline patches with C_n continuity across an edge is challenging.
- What happens to continuity at corners where the number of patches meeting isn't exactly four?
- Animation is tricky: bending and blending are doable, but not easy.



Sadly, the world isn't made up of shapes that can always be made from one smoothly-deformed rectangular surface.

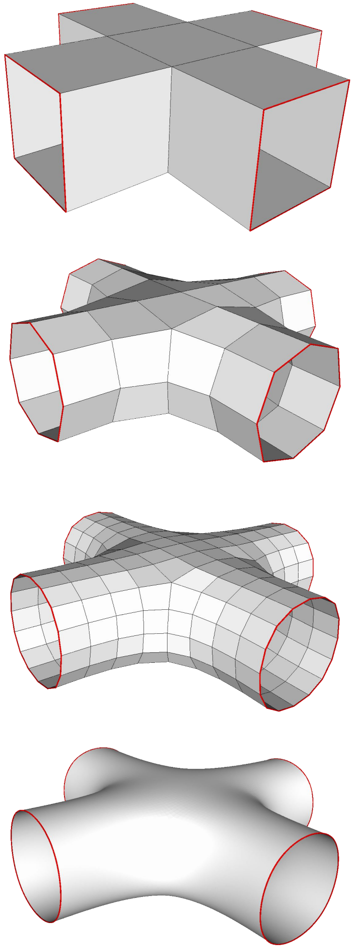
Subdivision surfaces

- Beyond shipbuilding: we want guaranteed continuity, without having to build everything out of rectangular patches.
  - Applications include CAD/CAM, 3D printing, museums and scanning, medicine, movies…
- The solution: *subdivision surfaces*.

Geri's Game, by Pixar (1997)

Subdivision surfaces

- Instead of ticking a parameter *t* along a parametric curve (or the parameters *u,v* over a parametric grid), subdivision surfaces repeatedly refine from a coarse set of *control points*.
- Each step of refinement adds new faces and vertices.
- The process converges to a smooth *limit surface*.



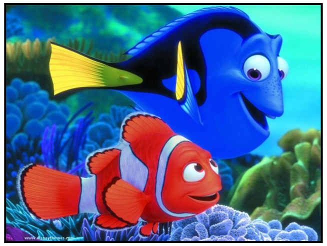
(Catmull-Clark in action)

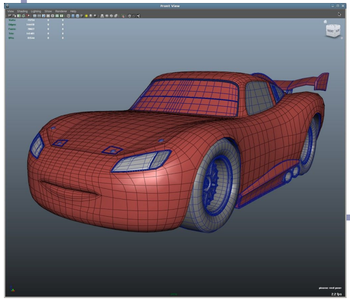
Subdivision surfaces – History

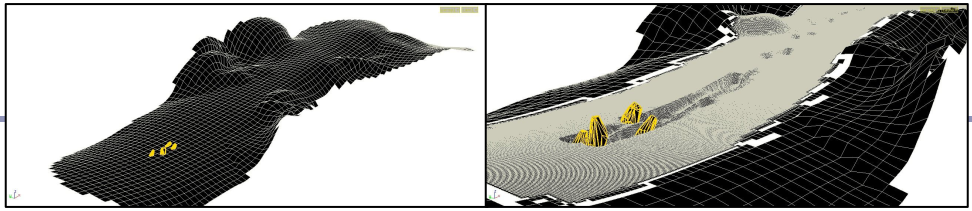
- de Rahm described a 2D (curve) subdivision scheme in 1947; rediscovered in 1974 by Chaikin
- Concept extended to 3D (surface) schemes by two separate groups during 1978:
  - Doo and Sabin found a biquadratic surface
  - Catmull and Clark found a bicubic surface
- Subsequent work in the 1980s (Loop, 1987; Dyn [Butterfly subdivision], 1990) led to tools suitable for CAD/CAM and animation

Subdivision surfaces and the movies

- Pixar first demonstrated subdivision surfaces in 1997 with Geri's Game.
  - Up until then they'd done everything in NURBS (Toy Story, A Bug's Life.)
  - From 1999 onwards everything they did was with subdivision surfaces (Toy Story 2, Monsters Inc, Finding Nemo...)
  - Two decades on, it's all heavily customized - creases and edges can be detailed by artists and regions of subdivision can themselves be dynamically subdivided

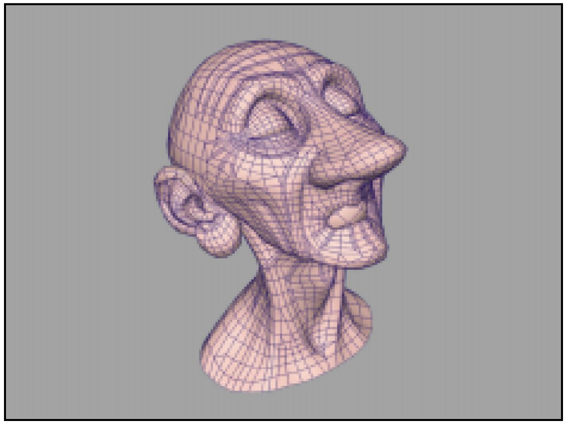






Useful terms

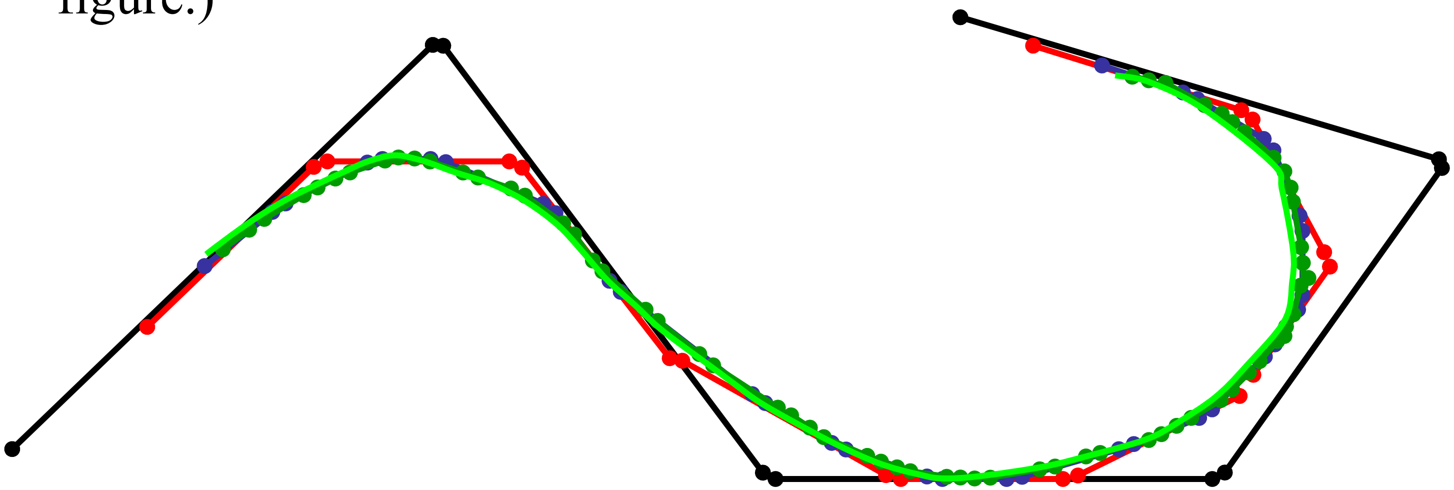
- A scheme which describes a 1D curve (even if that curve is travelling in 3D space, or higher) is called *univariate*, referring to the fact that the limit curve can be approximated by a polynomial in one variable (t).
- A scheme which describes a 2D surface is called *bivariate*, the limit surface can be approximated by a u,v parameterization.
- A scheme which retains and passes through its original control points is called an *interpolating* scheme.
- A scheme which moves away from its original control points, converging to a limit curve or surface nearby, is called an *approximating* scheme.



Control surface for Geri's head

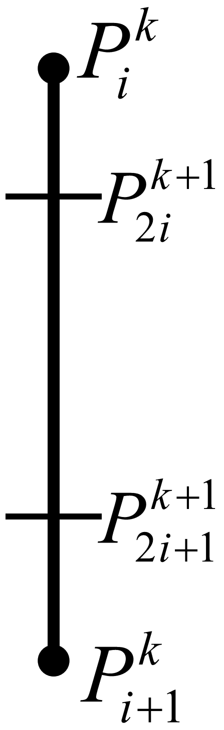
How it works

- Example: *Chaikin* curve subdivision (2D)
  - On each edge, insert new control points at ¼ and ¾ between old vertices; delete the old points
  - The *limit curve* is C1 everywhere (despite the poor figure.)



Notation

Chaikin can be written programmatically as:



$$P_{2i}^{k+1} = (\tfrac{3}{4})P_i^k + (\tfrac{1}{4})P_{i+1}^k \quad \leftarrow \textit{Even}$$

$$P_{2i+1}^{k+1} = (\tfrac{1}{4})P_i^k + (\tfrac{3}{4})P_{i+1}^k \quad \leftarrow \textit{Odd}$$

…where *k* is the ‘generation’; each generation will have twice as many control points as before.

Notice the different treatment of generating odd and even control points.

Borders (terminal points) are a special case.

Notation

Chaikin can be written in vector notation as:

$$\begin{bmatrix} \vdots \\ P_{2i-2}^{k+1} \\ P_{2i-1}^{k+1} \\ P_{2i}^{k+1} \\ P_{2i+1}^{k+1} \\ P_{2i+2}^{k+1} \\ P_{2i+3}^{k+1} \\ \vdots \end{bmatrix} = \frac{1}{4} \begin{bmatrix} & & & \vdots & & & \\ \cdots & 0 & 3 & 1 & 0 & 0 & 0 & \cdots \\ & 0 & 1 & 3 & 0 & 0 & 0 & \\ & 0 & 0 & 3 & 1 & 0 & 0 & \\ & 0 & 0 & 1 & 3 & 0 & 0 & \\ & 0 & 0 & 0 & 3 & 1 & 0 & \\ & 0 & 0 & 0 & 1 & 3 & 0 & \\ & & & \vdots & & & \end{bmatrix} \begin{bmatrix} \vdots \\ P_{i-2}^{k} \\ P_{i-1}^{k} \\ P_{i}^{k} \\ P_{i+1}^{k} \\ P_{i+2}^{k} \\ P_{i+3}^{k} \\ \vdots \end{bmatrix}$$

Notation

- The standard notation compresses the scheme to a *kernel*:
  - h =(1/4)[…,0,0,1,3,3,1,0,0,…]
- The kernel interlaces the odd and even rules.
- It also makes matrix analysis possible: eigenanalysis of the matrix form can be used to prove the continuity of the subdivision limit surface.
  - The details of analysis are fascinating, lengthy, and sadly beyond the scope of this course
- The limit curve of Chaikin is a quadratic B-spline!

Reading the kernel

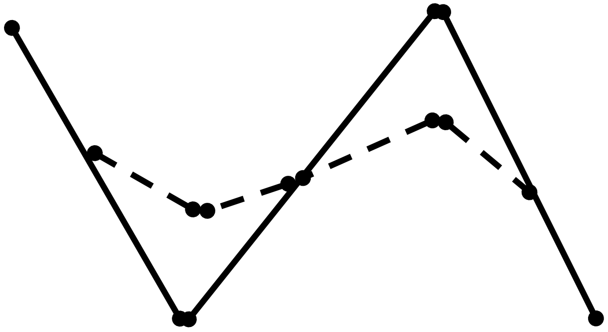
Consider the kernel

$$h=(1/8)[\ldots,0,0,1,4,6,4,1,0,0,\ldots]$$

You would read this as

$$P_{2i}^{k+1} = (1/8)(P_{i-1}^{k} + 6P_{i}^{k} + P_{i+1}^{k})$$

$$P_{2i+1}^{k+1} = (1/8)(4P_{i}^{k} + 4P_{i+1}^{k})$$

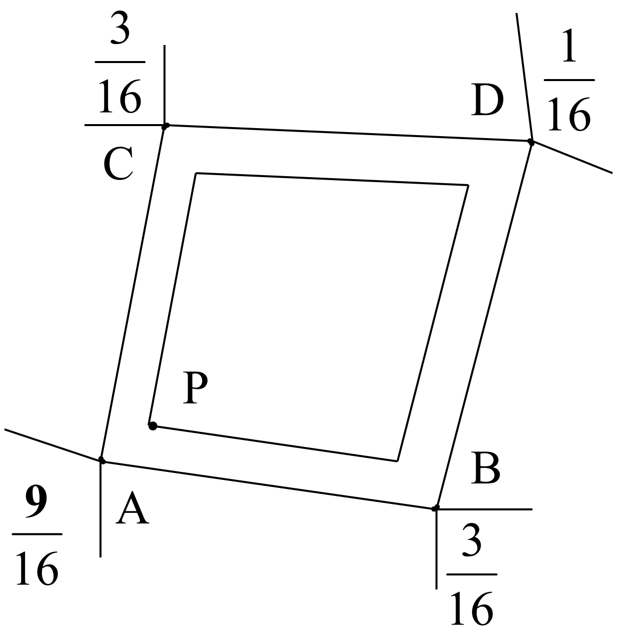


The limit curve is provably C2-continuous.

Making the jump to 3D: Doo-Sabin

Doo-Sabin takes Chaikin to 3D:

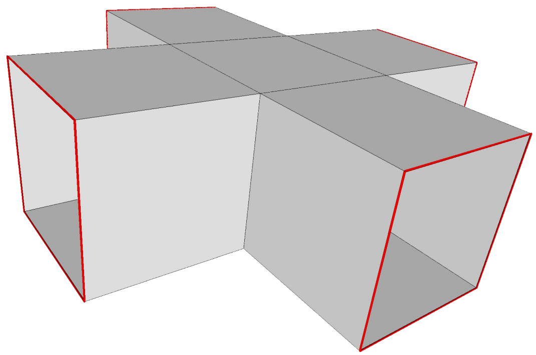
$$P = (9/16)\,A + (3/16)\,B + (3/16)\,C + (1/16)\,D$$



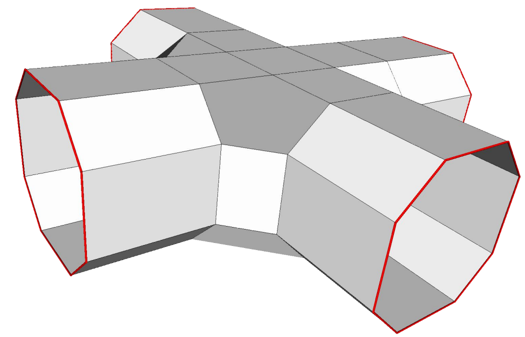
This replaces every old vertex with four new vertices.

The limit surface is biquadratic, C1 continuous everywhere.

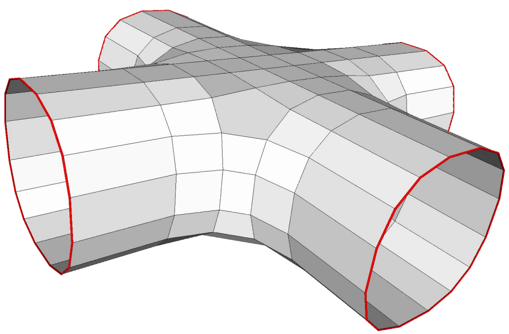
Doo-Sabin in action



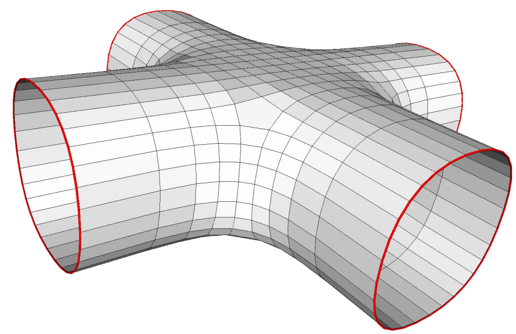
(0) 18 faces



(1) 54 faces



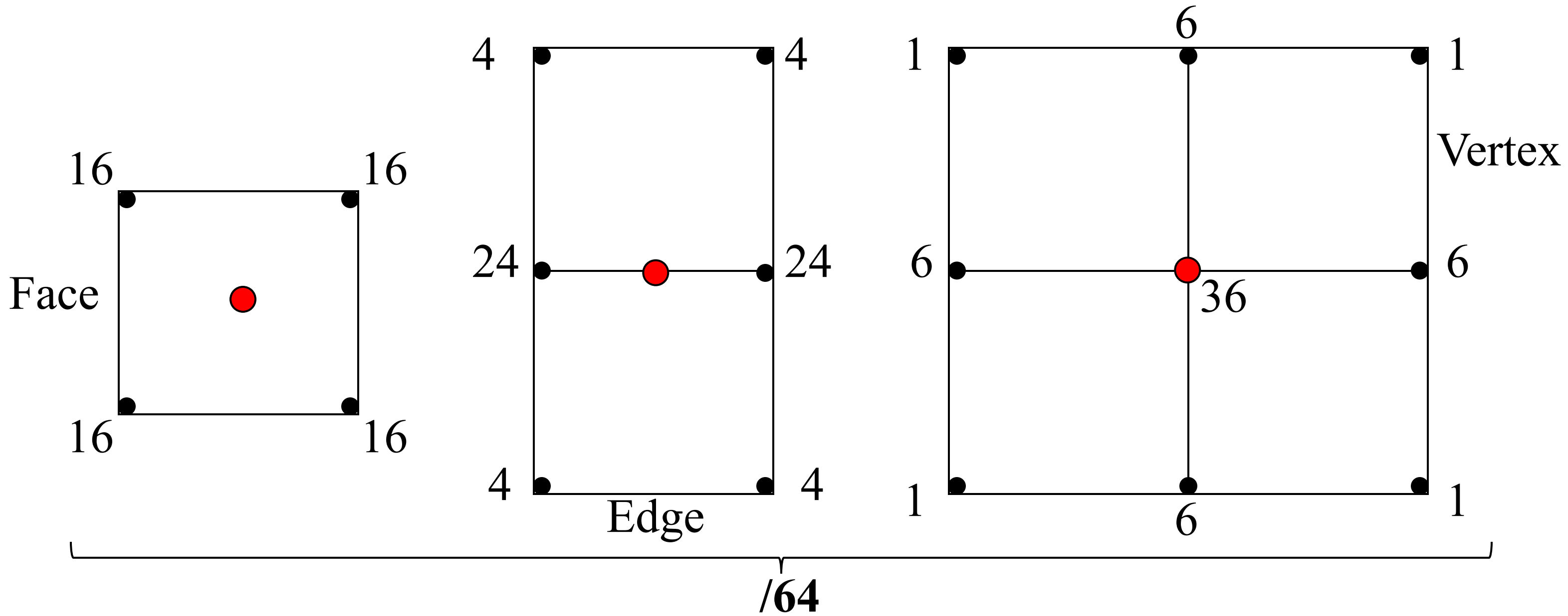
(2) 190 faces

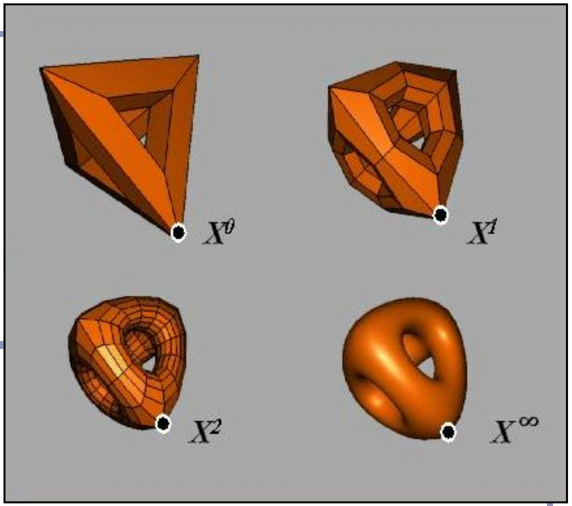


(3) 702 faces

Catmull-Clark

- *Catmull-Clark* is a bivariate approximating scheme with kernel h=(1/8)[1,4,6,4,1].
    - Limit surface is bicubic, C2-continuous.





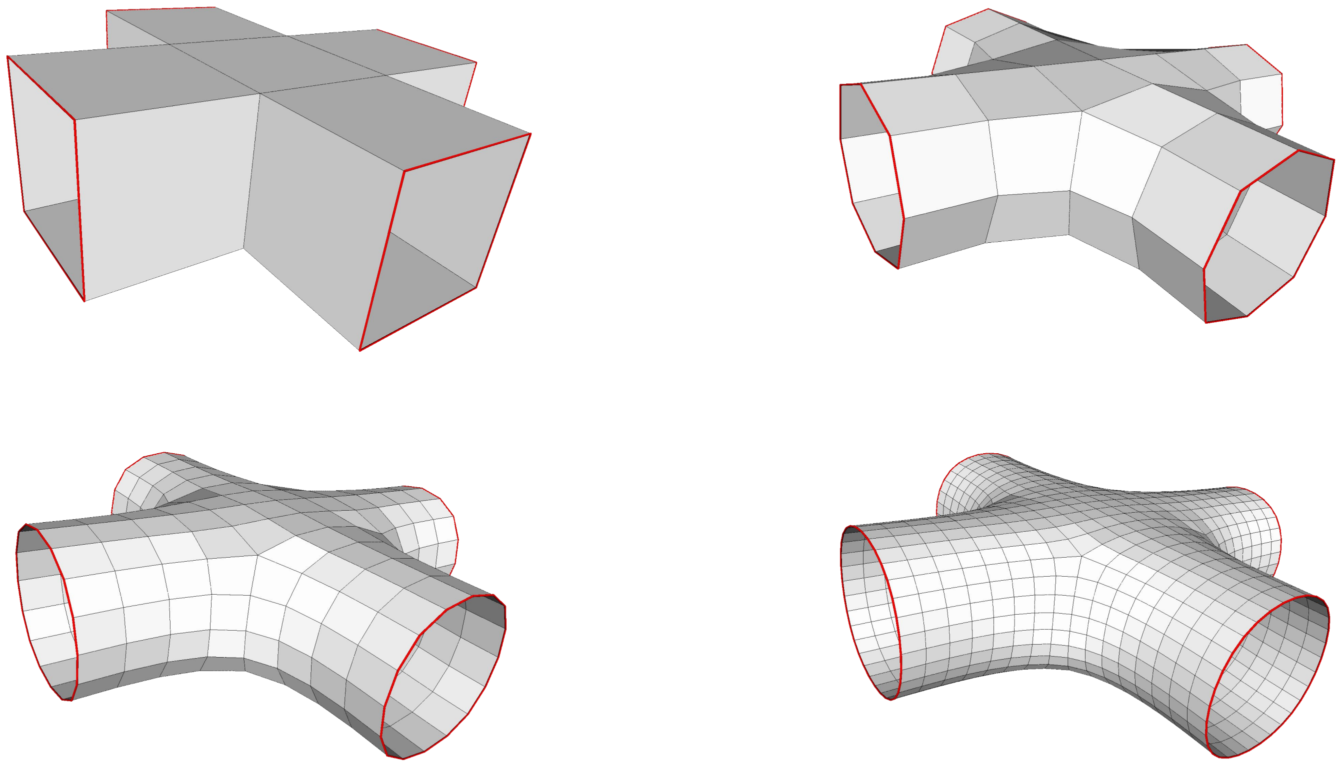
Catmull-Clark

Getting tensor again:

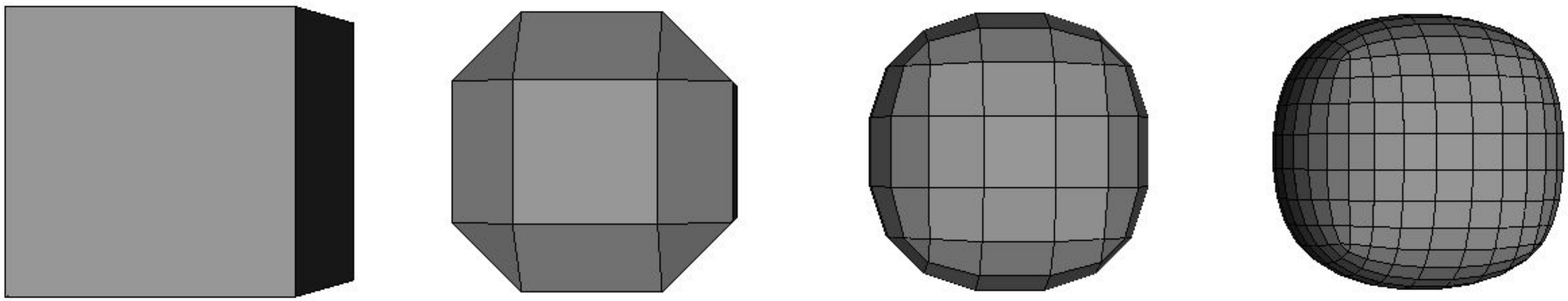
$$\frac{1}{8}\begin{bmatrix}1\\4\\6\\4\\1\end{bmatrix} \otimes \frac{1}{8}\begin{bmatrix}1\\4\\6\\4\\1\end{bmatrix} = \frac{1}{64}\begin{bmatrix}1 & 4 & 6 & 4 & 1\\4 & 16 & 24 & 16 & 4\\6 & 24 & 36 & 24 & 6\\4 & 16 & 24 & 16 & 4\\1 & 4 & 6 & 4 & 1\end{bmatrix}$$

Vertex rule Face rule Edge rule

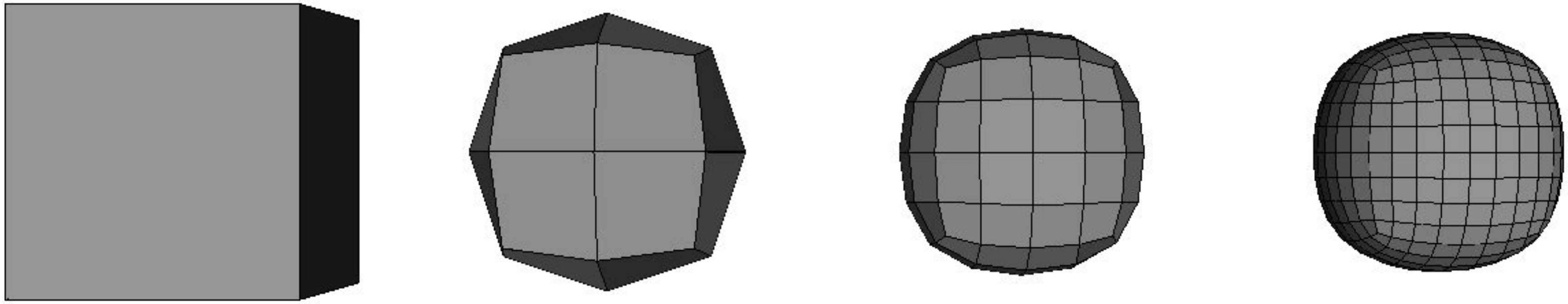
Catmull-Clark in action



Catmull-Clark vs Doo-Sabin



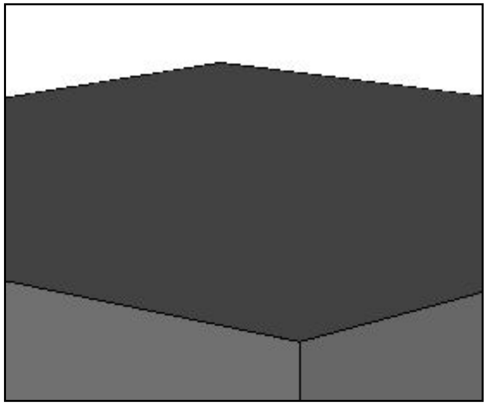
Doo-Sabin

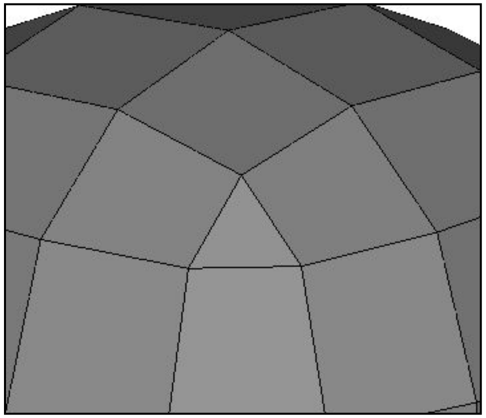


Catmull-Clark

Extraordinary vertices

- Catmull-Clark and Doo-Sabin both operate on quadrilateral meshes.
  - All faces have four boundary edges
  - All vertices have four incident edges
- What happens when the mesh contains *extraordinary* vertices or faces?
  - For many schemes, adaptive weights exist which can continue to guarantee at least some (non-zero) degree of continuity, but not always the best possible.
- CC replaces extraordinary faces with extraordinary vertices; DS replaces extraordinary vertices with extraordinary faces.





Detail of Doo-Sabin at cube corner

Extraordinary vertices: Catmull-Clark

Catmull-Clark vertex rules generalized for extraordinary vertices:

- Original vertex:

  $(4n\text{-}7) / 4n$
- Immediate neighbors in the one-ring:

  $3/2n^2$
- Interleaved neighbors in the one-ring:

  $1/4n^2$

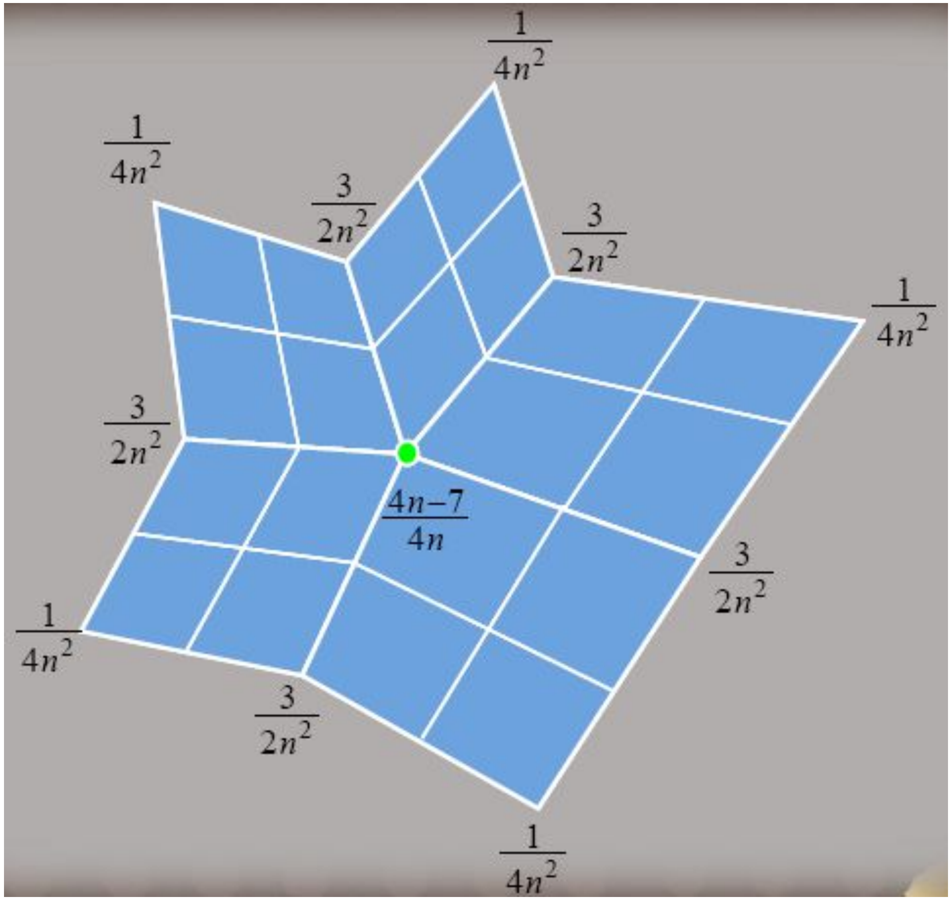
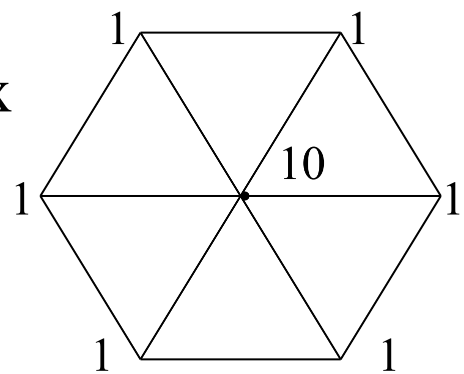


Image source: "*Next-Generation Rendering of Subdivision Surfaces*", Ignacio Castaño, SIGGRAPH 2008

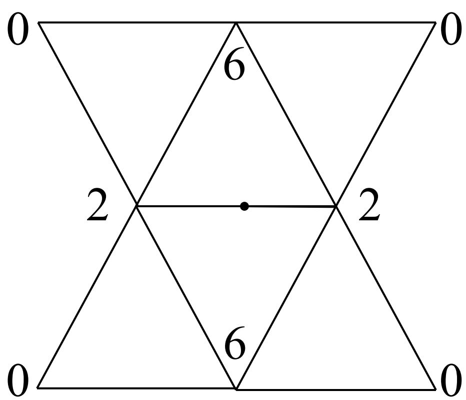
Schemes for simplicial (triangular) meshes

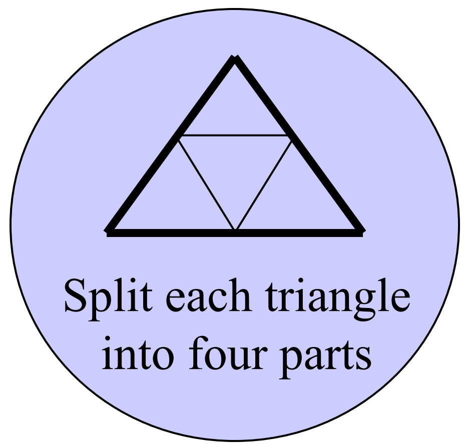
- *Loop* scheme:

Vertex



Edge

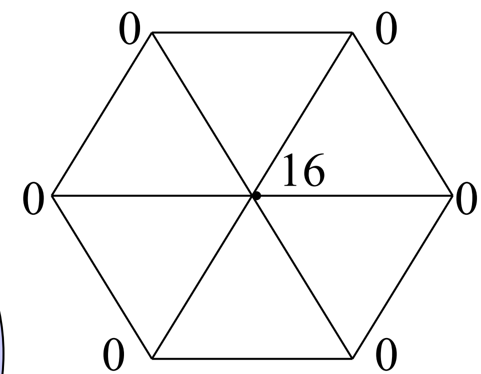




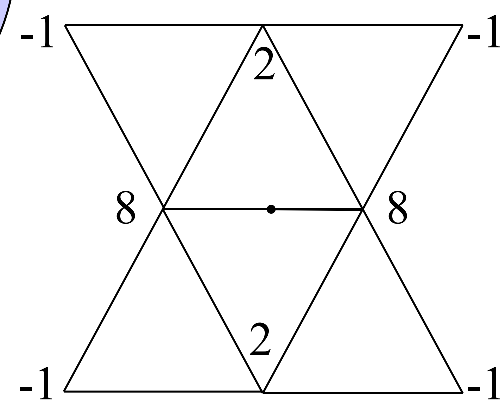
(All weights are /16)

- *Butterfly* scheme:

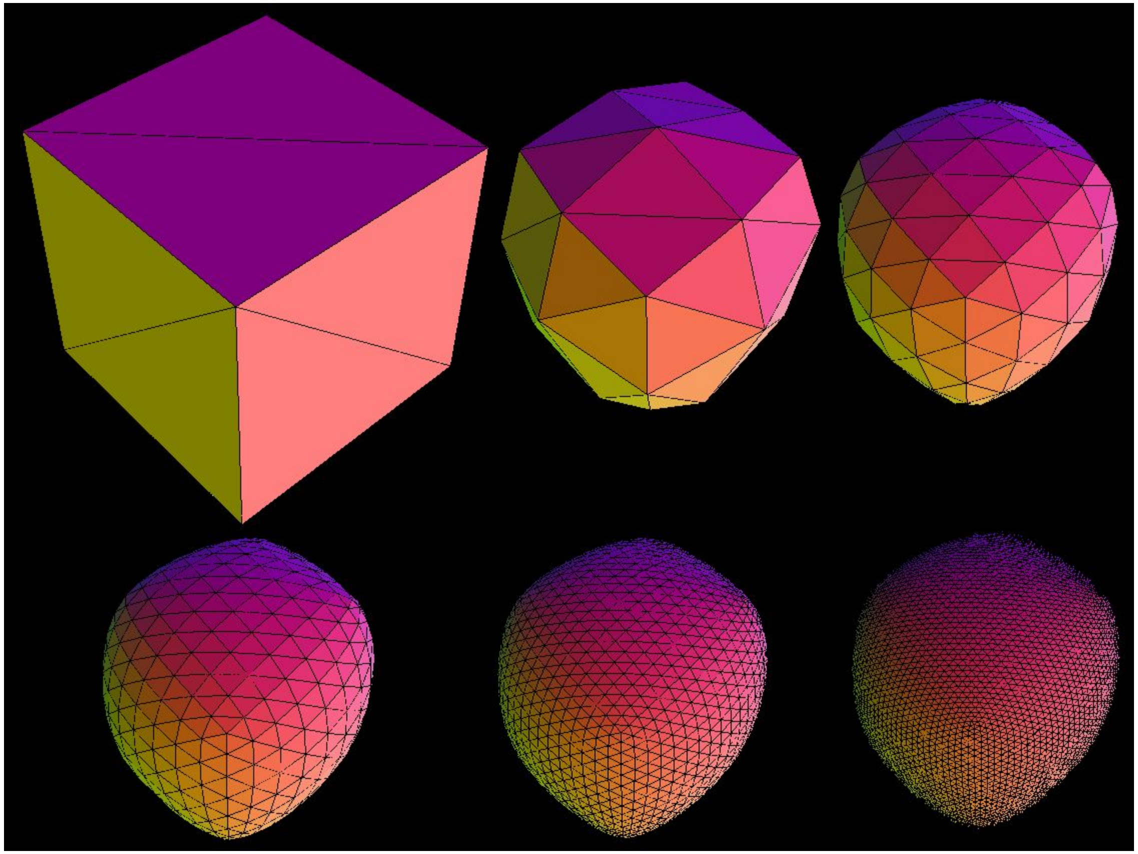
Vertex



Edge



Loop subdivision

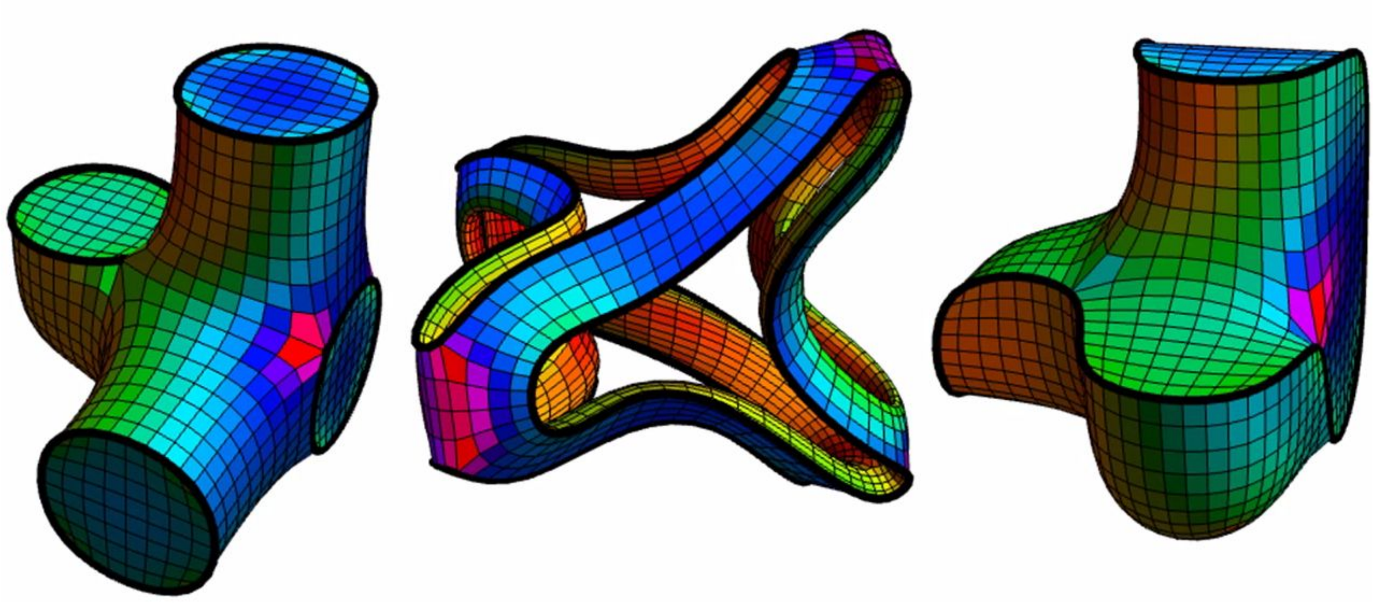


Loop subdivision in action. The asymmetry is due to the choice of face diagonals.

Image by Matt Fisher, http://www.its.caltech.edu/~matthewf/Chatter/Subdivision.html

Creases

Extensions exist for most schemes to support *creases*, vertices and edges flagged for partial or hybrid subdivision.



Still from "Volume Enclosed by Subdivision Surfaces with Sharp Creases" by Jan Hakenberg, Ulrich Reif, Scott Schaefer, Joe Warren http://vixra.org/pdf/1406.0060v1.pdf

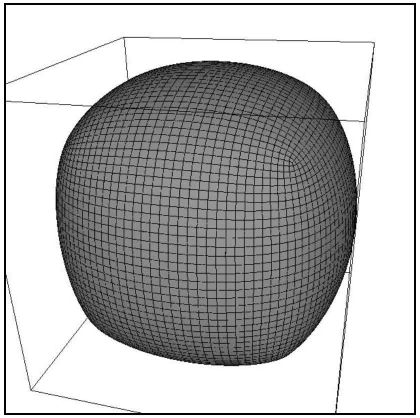
Splitting a subdivision surface

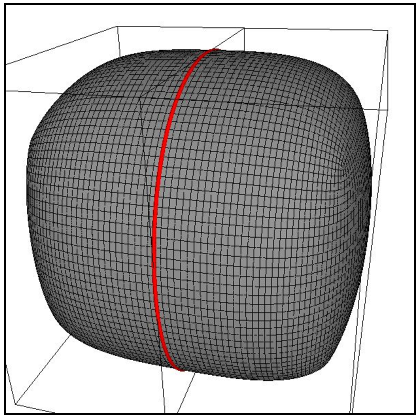
Many algorithms rely on subdividing a surface and examining the bounding boxes of smaller facets.

- Rendering, ray/surface intersections…

It's not enough just to delete half your control points: the limit surface will change (see right)

- Need to include all control points from the previous generation, which influence the limit surface in this smaller part.

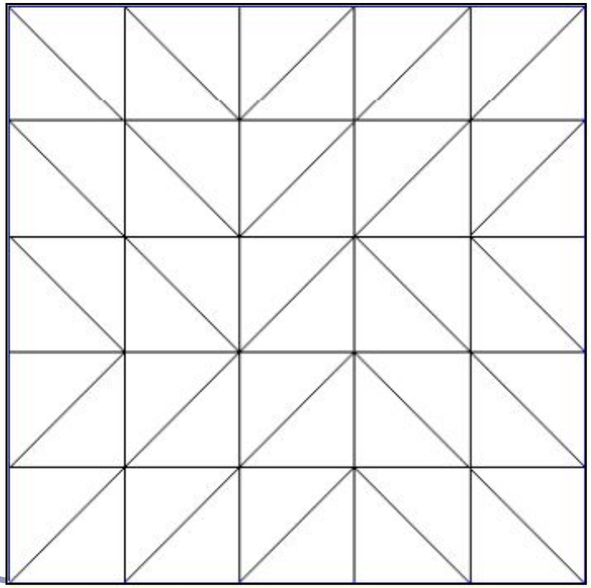




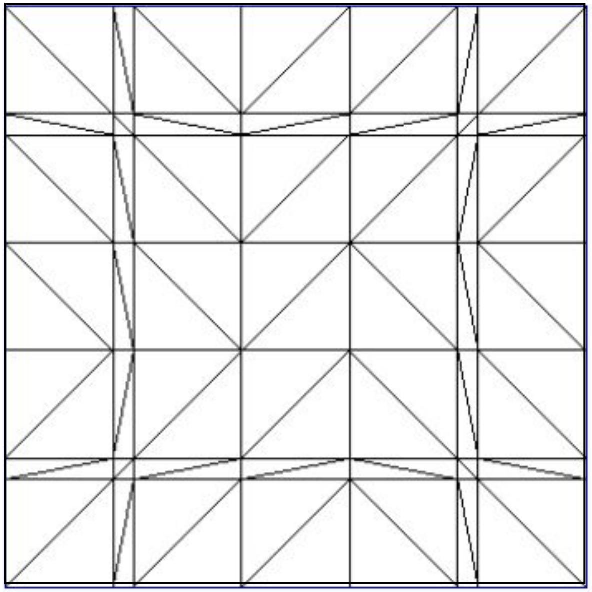
(Top) 5x Catmull-Clark subdivision of a cube
(Bottom) 5x Catmull-Clark subdivision of two halves of a cube; the limit surfaces are clearly different.

Continuous level of detail

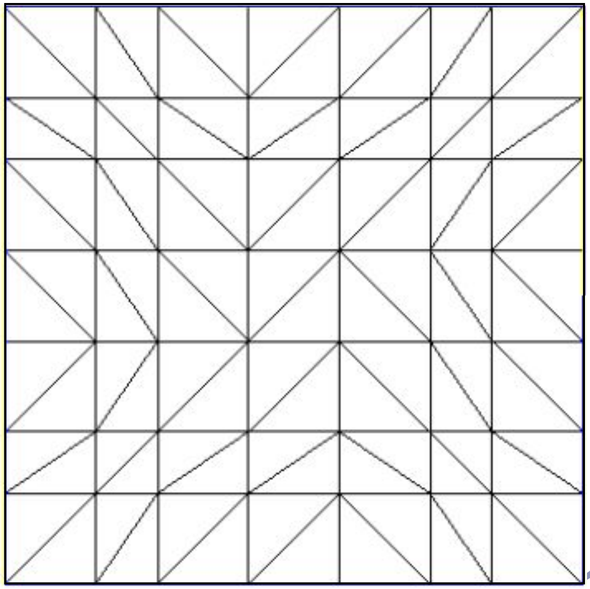
For live applications (e.g. games) can compute *continuous* level of detail, typically as a function of distance:



Level 5



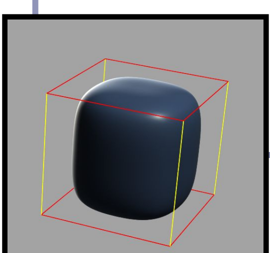
Level 5.2



Level 5.8

Bounding boxes and convex hulls for subdivision surfaces

- The limit surface is (the weighted average of (the weighted averages of (the weighted averages of (repeat for eternity…)))) the original control points.
- This implies that for any scheme where all weights are positive and sum to one, the limit surface lies entirely within the convex hull of the original control points.
- For schemes with negative weights:
  - Let $L=max_t \Sigma_i |N_i(t)|$ be the greatest sum throughout parameter space of the absolute values of the weights.
  - For a scheme with negative weights, L will exceed 1.
  - Then the limit surface must lie within the convex hull of the original control points, expanded unilaterally by a ratio of (L-1).



Subdivision Schemes—A partial list

- Approximating
  - Quadrilateral
    - (1/2)[1,2,1]
    - (1/4)[1,3,3,1] (Doo-Sabin)
    - (1/8)[1,4,6,4,1] (Catmull-Clark)
    - *Mid-Edge*
  - Triangles
    - Loop
- Interpolating
  - Quadrilateral
    - *Kobbelt*
  - Triangle
    - Butterfly
    - *"√3" Subdivision*

Many more exist, some much more complex

This is a major topic of ongoing research

References

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