Exercises for Advanced Graphics (Lectures 1-4)

All work to be submitted by email in PDF format, no less than 48 hours before supervision.

These exercises are partly drawn from past exam questions.

1. **B-Splines and Beziers**
    a. Show that the B-spline with k=3 and knot vector [0 0 0 1 1 1] is equivalent to the quadratic Bezier curve.
    b. Give a knot vector and value of k which would describe a uniform Bspline equivalent to a cubic Bezier curve.

2. **B-splines**
    a. Derive the formula of and sketch a graph of N3,3(t), the third of the quadratic B-spline basis functions, for the knot vector [0 0 0 1 3 3 4 5 5 5].
    b. We have already seen that, for a given order, k, there is only one basis function for uniform B-splines. Every control point uses a shifted version of that one basis function. How many different basis functions are there for open-uniform B-splines of order k=2? Order k=3? For the general case, that is: for arbitrary k?
    c. Imagine a type of B-spline where adjacent knots are separated by either 1 or 0. This gives uniform knot separation with the added possibility of multiple knots. e.g. (0,0,0,1,2,3,3,3); (0,0,1,1,2,2); (0,1,1,1,1,2,2,3,4,4).How many different basis functions will be needed for this type of B-spline for k=2? For k=3? For arbitrary k? (Hint: you may like to start by considering the case k=1).

3. **Bezier patches**
    Give the coefficient polynomials for a bivariate quadratic triangular Bezier patch. This was not covered in lecture: you will have to do a little research. (Or derive them from first principles, of course.) Be sure that your answer is truly bivariate (only two varying parameters) and please cite your sources where appropriate.

4. **Doo-Sabin and Reif-Peters Subdivision**
    The Reif-Peters subdivision scheme is illustrated at right. Reif-Peters uses a different approach to Doo-Sabin: in it the new points are generated halfway between existing points and connected up into a mesh as illustrated in the

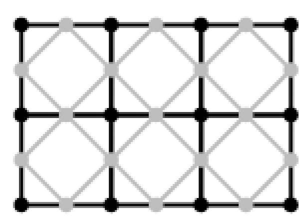
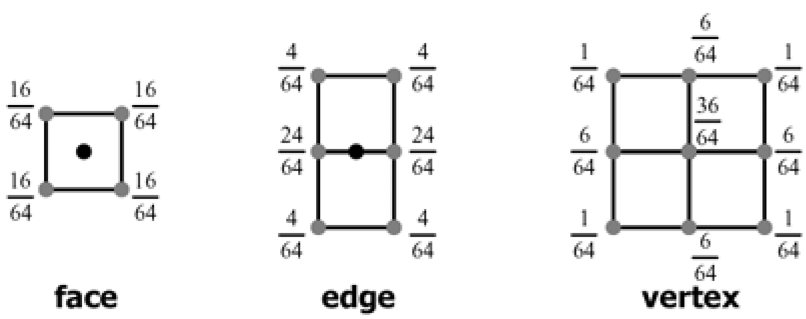


diagram on the right (black: original mesh; grey: new mesh). Note that (i) the existing points do not form part of the new mesh and (ii) each new point's position is simply the average of the positions of the two existing points at either end of the corresponding line segment.

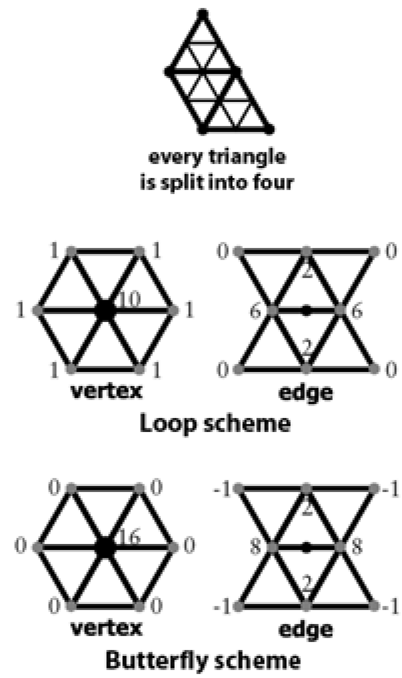
a. Doing the Reif-Peters subdivision twice produces a mesh which looks similar to that produced by doing the Doo-Sabin subdivision once. You can treat two steps of Reif-Peters as if it were a single step of a Doo-Sabin-like subdivision scheme. Calculate the weights on the original points for each new point after two steps of Reif-Peters.
b. For the Reif-Peters scheme, explain what happens around extraordinary vertices and what happens around extraordinary polygons, giving examples.

5. **Catmull-Clark Subdivision**
The Catmull-Clark bivariate subdivision scheme is a bivariate generalisation of the univariate 1/8 [1, 4, 6, 4, 1] subdivision scheme. It creates new vertices as blends of old vertices in the following ways:



a. Provide similar diagrams for the bivariate generalisation of the univariate four-point interpolating subdivision scheme 1/16 [-1, 0, 9, 16, 9, 0,-1].
b. Explain what problems arise around extraordinary vertices (vertices of valency other than four) for this bivariate interpolating scheme and suggest a possible way of handling the creation of new edge vertices when the old vertex at one end of the edge has a valency other than four.

6. **Triangular Subdivision**
The Loop and Butterfly subdivision schemes can both operate on triangular meshes, in which all of the polygons have three sides. Both schemes subdivide the mesh by introducing new vertices at the midpoints of edges, splitting every original triangle into four smaller triangles, as shown at right. Each scheme has rules for calculating the locations of the new "edge" and "vertex" vertices based on the locations of the old vertices. These rules are shown below. All weights should be multiplied by 1/16.



a. Which of the two schemes produces a limit surface which interpolates the original data points?
b. Which of the four rules must be modified when there is an extraordinary vertex? For each of the four rules either explain why it must be modified or explain why it does not need to be modified.
c. Suggest appropriate modifications where necessary to accommodate extraordinary vertices.

7. **Implicit Surfaces**
    a. Explain the special cases in the polygonalization of an octree, and how you might address them.
    b. Summarize the *marching cubes* algorithm.

8. **Voronoi Diagrams**
    a. What is *equiangularity*?
    b. What is the *empty circle property*?

9. **Curvature**
    The *one-ring* of a vertex is the (usually ordered) set of vertices which lie exactly one edge away from a given vertex on a polyhedral surface. Given a vertex V with one-ring $\{v_0,...,v_{n-1}\}$, give a formula for the discrete curvature of the surface at V.