Exercises for Neil Dodgson's Lectures (1/2)

All work must be submitted by email no less than 48 hours before supervision.
These exercises are drawn from past exam questions.

1) B-Splines and Beziers
a) Show that the B-spline with k=3 and knot vector [0 0 0 1 1 1] is equivalent to the quadratic Bezier curve.
b) Give a knot vector and value of k which would describe a uniform Bspline equivalent to a cubic Bezier curve.

2) B-splines
a) Derive the formula of and sketch a graph of N3,3(t), the third of the quadratic B-spline basis functions, for the knot vector [0 0 0 1 3 3 4 5 5 5].
b) We have already seen that, for a given order, k, there is only one basis function for uniform B-splines. Every control point uses a shifted version of that one basis function. How many different basis functions are there for open-uniform B-splines of order k=2? Order k=3? For the general case, that is: for arbitrary k?
c) Imagine a type of B-spline where adjacent knots are separated by either 1 or 0. This gives uniform knot separation with the added possibility of multiple knots. e.g. (0,0,0,1,2,3,3,3); (0,0,1,1,2,2); (0,1,1,1,1,2,2,3,4,4).How many different basis functions will be needed for this type of B-spline for k=2? For k=3? For arbitrary k?
(Hint: you may like to start by considering the case k=1).

3) NURBS – understanding homogeneous coordinates
Use the quarter-circle definition from the notes.
a) Graph what happens for at least five values of α. Amongst the values that you use, include the cases α=0, α=1, and α→∞.
b) Consider the line y=x. For any given value of α, the NURBS curve from equation (34) will cut the line y=x in one place, call it (b,b). Find the mathematical relationship between α and b.

4) Bezier patches
Give the coefficient polynomials for a bivariate quadratic triangular Bezier patch. This was not covered in lecture: you will have to do a little research. (Or derive them from first principles, of course.) Be sure that your answer is truly bivariate (only two varying parameters) and please cite your sources where appropriate.